

$$10) \quad ax^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \quad \swarrow \alpha, \beta \text{ terms}$$

$$ax^2 + bx + c = a(x - \alpha)(x - \beta)$$

$$ax^2 + bx + c = a \{ x^2 - (\alpha + \beta)x + \alpha\beta \}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$1) \text{SOR} = \alpha + \beta = -\frac{b}{a}$$

$$2) \text{POR} = \alpha\beta = \frac{c}{a}$$

$$3) \text{Difference of Root} = \text{DOR} = |\alpha - \beta| \\ = \sqrt{(\alpha - \beta)^2}$$

$$\text{DOR} = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$= \sqrt{\frac{b^2}{a^2} - 4\frac{c}{a}}$$

$$= \frac{\sqrt{b^2 - 4ac}}{|a|}$$

$$\text{DOR} = |\alpha - \beta| = \frac{\sqrt{D}}{|a|}$$

$$\ast \text{R.K.} \rightarrow \text{If } a + b + c = 0 \rightarrow \alpha = 1$$

$$\text{B) If roots are in cyclic order} \\ (a-b)x^2 + (b-c)x + (c-a) = 0 \rightarrow \alpha = 1$$

$$x^2 - Sx + P = 0$$

Q Form a Q Eqn. With Rational coeff
Whose 1 Root is $\tan \frac{\pi}{8} = (\sqrt{2}-1)$

$$\alpha = \tan \frac{\pi}{8} = \sqrt{2}-1 \quad \left\{ \begin{array}{l} \text{Walek} \\ \text{Satn sign change} \end{array} \right.$$

then $\beta = -\sqrt{2}-1$

$$x^2 - (\sqrt{2}-1 + -\sqrt{2}-1)x + (\sqrt{2}-1)(-\sqrt{2}-1) = 0$$

$$x^2 + 2x + ((-1)^2 - (\sqrt{2})^2) = 0$$

$$x^2 + 2x - 1 = 0$$

Q Find value of m if eqn.

$$x^2 - (m+2)x + (m^2 - 4m + 4) = 0$$

has coincident Roots?

Similar $\rightarrow D=0$

$$(m+2)^2 - 4 \times 1 \times (m^2 - 4m + 4) = 0$$

$$m^2 + 4m + 4 - 4m^2 + 16m - 16 = 0$$

$$-3m^2 + 20m - 12 = 0$$

$$3m^2 - 20m + 12 = 0$$

$$(3m-2)(m-6) = 0$$

$$m = \frac{2}{3}, 6$$

$$x = \frac{-b + \sqrt{D}}{2a}, \frac{-b - \sqrt{D}}{2a}$$

$$a, b, c \in \mathbb{Q}$$

$$D = b^2 - 4ac$$

$$2+i, 2-i$$

$$x = \frac{-b \pm \sqrt{-ve}}{2a}$$

$$\sqrt{24}$$

$$2\sqrt{6}$$

$$2\sqrt{2}\sqrt{3}$$

Ir Ir

$$D=0$$

Similar Root
(α, α)

$$a(x-\alpha)(x-\alpha)$$

$$= a(x-\alpha)^2 \rightarrow \text{Per sq}$$

$$= a\left(x + \frac{b}{2a}\right)^2$$

$$D \neq 0$$

$$D > 0 = \underline{24, 25}$$

$$\text{Per sq}$$

$$D=25$$

Rational
Roots

$$\frac{-b+5}{2a}, \frac{-b-5}{2a}$$

Distinct
Roots

$$D \neq \text{Per sq}$$

Irr Root
Boho

$$P + \sqrt{q}, P - \sqrt{q}$$

Conjugate Pair

$$a=1, b, c = \text{Integers}$$

$$D = \text{Per sq}$$

$$\alpha, \beta = \text{Integers}$$

Root Imaginary
 $P+iq, P-iq$
BOHO offer.

Q Find value of K for which
a Quad Poly $f(x) =$
 $(4-K)x^2 + (2K+4)x + (8K+1)$
is a Per Sq Find value of K ?
 $\hookrightarrow D=0$

$$(2K+4)^2 - 4 \times (4-K)(8K+1) = 0$$

$$(K+2)^2 - (-8K^2 + 31K + 4) = 0$$

$$K^2 + 4K + 4 + 8K^2 - 31K - 4 = 0$$

$$9K^2 - 27K = 0$$

$$9K(K-3) = 0 \rightarrow K = 0, 3$$

$$Q \text{ Per Sq is } a? \rightarrow \left(x + \frac{b}{2a}\right)^2 - 0 = 0$$

Q If Eqⁿ $P(q-r)x^2 + q(r+p)x + r(p-q) = 0$
has eq² Roots then $\frac{2}{q} = ?$

Coeff are in cyclic Order.

$\Rightarrow$ One Root is $1 \rightarrow$ 2nd is 1

$$PQR = \frac{r(p-q)}{p(q-r)} = 1$$

$$pr - qr = pq - pr$$

$$2pr = q(p+r)$$

$$\frac{2}{q} = \left[\frac{1}{r} + \frac{1}{p} \right] Q_{11}$$

$$D = \text{PerSq}^r \rightarrow \text{RationalRoots}$$

Q quad Eqⁿ in the form of PerSq^r.

$$\Rightarrow D = 0$$

Q. Q Eqⁿ $(m^2 - 3)x^2 + 3mx + (3m + 1) = 0$

eqⁿ has ReciprocalRoots for all m ?

1 Root α then other other $= \frac{1}{\alpha}$

$$\text{POR} = \alpha \times \frac{1}{\alpha} = 1$$

$$\frac{3m+1}{m^2-3} = 1 \Rightarrow m^2 - 3 = 3m + 1$$

$$m^2 - 3m - 4 = 0$$

$$(m-4)(m+1) = 0$$

$$m = 4, -1$$

Q If α, β are Roots of Q Eqⁿ

$ax^2 + bx + c = 0$ then find

(1) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (2) $\alpha^3 + \beta^3$

$$\frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{\left(-\frac{b}{a}\right)^2 - 2\frac{c}{a}}{\frac{c}{a}}$$

$$\begin{aligned}
 (2) \quad a^3 + b^3 &= (a+b)(\underbrace{a^2 + b^2}_{-2ab}) - \underbrace{ab}_{-ab} \\
 &= (a+b)(a+b)^2 - 2ab - ab \\
 &= (a+b)(a+b)^2 - 3ab \\
 &= \left(-\frac{b}{a}\right) \left(\frac{b^2}{a^2} - \frac{3b}{a}\right)
 \end{aligned}$$

Q. $x^2 - (2\sqrt{2})x + 1 = 0$ $b = 2\sqrt{2} \notin \mathbb{Q}$ Nature of Roots?

find $D = (2\sqrt{2})^2 - 4 \times 1 \times 1 = 8 - 4 = +ve = 4$ Rational

$$x = \frac{2\sqrt{2} \pm \sqrt{8-4}}{2} = \frac{2\sqrt{2} \pm 2}{2} = \frac{\sqrt{2} \pm 1}{1}, \sqrt{2} - 1$$

Irr. Roots

Q. $x^2 - 2(a+b)x + 2(a^2 + b^2) = 0$
 HW Nature of Roots? Imag

A) Both Roots +ve $\rightarrow \alpha, \beta \oplus$ A) $D \geq 0$ (B) $\alpha + \beta > 0$ (C) $\alpha \cdot \beta > 0$

B) Both Roots -ve $\rightarrow \alpha, \beta \ominus$ A) $D \geq 0$ (B) $\alpha + \beta < 0$ (C) $\alpha \cdot \beta > 0$

(C) Roots opp in Sign A) $D > 0$ (B) $\alpha \neq \beta$ (C) $\alpha \cdot \beta < 0$

(C) Roots opp in Sign
 $\alpha + \beta - 2, \alpha - \beta +$

Exactly one Root +ve & other -ve

(D) Roots opp in Sign $\geq \text{eq}^L \rightarrow$ A) $D > 0$ (B) $\alpha + \beta = 0$ (C) $\alpha \cdot \beta < 0$

$\alpha + \alpha -$
 $2, -2$

(E) At least one Root +ve

Exactly one Root +ve $\rightarrow$ (C)
 Both root +ve $\rightarrow$ A

A U C

At least 1 Root -ve $\left\{ \begin{array}{l} \rightarrow \text{Exactly one Root -ve} \rightarrow C \\ \rightarrow \text{Both Root -ve} \rightarrow B \end{array} \right\} \boxed{B \cup C}$

MASTER PROBLEM.

Q Find Range of m for which (1) Eqⁿ $x^2 - (m-3)x + m = 0$ has

A) Both Imaginary Roots $\rightarrow D < 0 \Rightarrow \frac{(m-3)^2 - 4 \times 1 \times m}{1} < 0$
 $\Rightarrow m^2 - 6m + 9 - 4m < 0 \Rightarrow m^2 - 10m + 9 < 0$
 $\Rightarrow (m-1)(m-9) < 0 \Rightarrow \boxed{1 < m < 9}$

(B) Both roots equal $\rightarrow D = 0 \Rightarrow (m-1)(m-9) = 0 \Rightarrow m = 1, 9$.

(C) Both Roots Real & Distinct $\rightarrow D > 0 \Rightarrow (m-1)(m-9) > 0 \Rightarrow m < 1 \vee m > 9$

(D) Both +ve Roots A) $D \geq 0 \rightarrow m \leq 1 \vee m \geq 9$
 $\frac{-(m-3)}{1} > 0 \Rightarrow m < 3$
 $\frac{m}{1} > 0 \Rightarrow m > 0$
 $m \in [9, \infty)$

(E) Both Roots -ve $\rightarrow D \geq 0$

$(m-1)(m-9) \geq 0$
 $m \leq 1 \cup m \geq 9$

$\alpha + \beta < 0$
 $\frac{(m-3)}{1} < 0$
 $m < 3$

$\alpha \cdot \beta > 0$
 $\frac{m}{1} > 0$
 $m > 0$

$m \in (0, 1]$

(F) Roots opp in Sign. } (i) $D > 0$

Exactly 1 Root +ve } $(m-1)(m-9) > 0$
 $m < 1 \cup m > 9$

$\alpha + \beta < 0$
 $\frac{m}{1} < 0$
 $m < 0$



$$\Delta, -B \rightarrow \Delta, -\Delta$$

(7) Roots Equal in Magnitude & opp in Sign ① $D > 0$ ② $\Delta + B = 0$ ③ $\Delta < B < 0$

Isi Tgh

Se start Kareenge

practice
as many Qs
as possible

$$m < 1 \cup m > 9 \quad | \quad \underline{m = 3}, \quad | \quad m < 0$$



$$0 \quad 1 \quad 3 \quad 9$$

(8) At least one Root +ve = { exactly 1 Root +ve $\cup$ Both Root +ve }
 $(-\infty, 0) \cup [9, \infty)$

$m = 0$ (check) $\rightarrow x^2 + 3x = 0$

$$\boxed{x = 0}, \quad \boxed{x = -3}$$

$\Delta_2 = 144$

$m = 9$ (check)

$$a, b, c \in \mathbb{Q}$$

A) Distinct Roots $\rightarrow D > 0, D < 0$

B) Real & Distinct Roots $\rightarrow D > 0$

C) Equal Root $\rightarrow D = 0$

D) Real Roots $\rightarrow D \geq 0$

E) Conjugate Pairs $\rightarrow$ Imaginary $(P+iq, P-iq)$
 $\rightarrow$ Irr. Roots $\rightarrow (P+\sqrt{q}, P-\sqrt{q})$

(F) Ration Roots $\rightarrow D = \text{Perfect sq}^r$

(G) Irr Roots $\rightarrow D \neq \text{Per sq}^r$